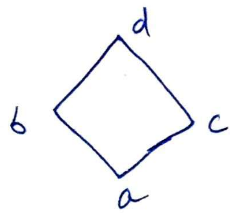
12. Types of ~~Sub~~lattice :-

1. Complete Lattice
2. Bounded Lattice
3. Isomorphic Lattice
4. Distributive Lattice
5. Complemented Lattice

1. Complete Lattice :- A lattice P is called Complete iff Every non-empty subset of P has g.l.b & l.u.b.

e.g.

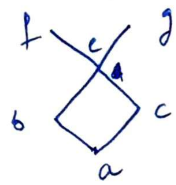
1. 

g.l.b = a

l.u.b = d

P = {a, b, c, d}

S = {b, c}

2. 

It is not lattice so it is not Complete lattice.

∵ f, g has not upper bound.

Every finite lattice is Complete.

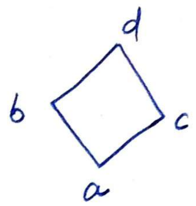
2. Bounded Lattice :-

Bounds :- The least and greatest elements of a lattice are called as bounds.

Least Element is denoted by 0,

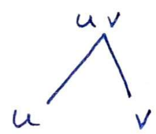
Greatest Element is denoted by 1.

e.g.

1. 

0 = a

1 = d

2. 

1 = uv

0 = None.

Bounded Lattice is a Lattice which has both Elements 0 & 1 is called as bdd lattice.

Denoted by $(L, \vee, \wedge, 0, 1)$

$x \vee y =$ l.u.b $\{x, y\}$ (Join of x & y)

$x \wedge y =$ g.l.b $\{x, y\}$ (meet of x & y)

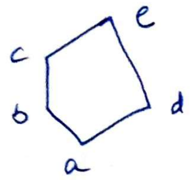
* $0 \wedge a = 0 = a \wedge 0$ $\quad \forall a \in L$

$0 \vee a = a = a \vee 0$

$1 \wedge a = a = a \wedge 1$

$1 \vee a = 1 = a \vee 1$

e.g.

1. 

Here $0 = a$

$1 = e$

$\therefore$ it is bdd lattice.

2. $(Z, \leq)$ - Not bdd lattice

$\because$ it has neither least Element (0) & nor Greatest Element (1)

3. $(Z^+, |)$ - Not bdd lattice.

$\because$ it has a least Element but no greatest Element.

3. Isomorphic Lattice: Let L_1 & L_2 be two lattice

$L_1 \cong L_2$ iff $\exists$ a bijection (1-1, onto) from L_1 to L_2

s.t. $f(a \wedge b) = f(a) \wedge f(b)$

$f(a \vee b) = f(a) \vee f(b)$

e.g 1. Consider $L = \{1, 2, 3, 4, 5, 6\}$

mapping $f : (L, |) \rightarrow (L, \leq)$

Let $2, 3 \in L$

$2 \vee 3 = \text{l.cm}\{2, 3\}$

$= 6$

L.H.S $f(2 \vee 3) = f(6)$

R.H.S $f(2) \vee f(3) = f(2) \text{ or } f(3)$

$\therefore f(2) \text{ or } f(3) \neq f(6)$

L.H.S $\neq$ R.H.S.

$\therefore$ It is not Isomorphic Lattice

e.g 2. Let $L = \{1, 2, 3, 6\}$ and $A = \{a, b\}$ Then P.T. Lattices $(L, |)$ & $(P(A), \subseteq)$ are isomorphic.

Sol: Consider mapping $f : L \rightarrow P(A)$

$P(A) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

$\therefore f(1) = \phi$

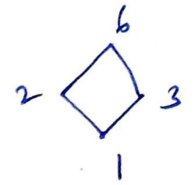
$f(2) = \{a\}$

$f(3) = \{b\}$

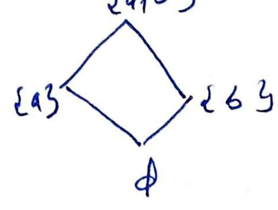
$f(6) = \{a, b\}$

Hasse Diagram of

$L \Rightarrow$



$P(A) \Rightarrow$



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Now Consider $1, 2 \in L$

L.H.S $f(1 \wedge 2) = f(1) = \phi$

R.H.S $f(1) \wedge f(2) = \phi \wedge \{a\} \Rightarrow \phi$

L.H.S = R.H.S

Now $f(1 \vee 2) = f(2) = \{a\}$

& $f(1) \vee f(2) = \phi \vee \{a\} = \{a\}$

Hence $f(1 \wedge 2) = f(1) \wedge f(2)$

& $f(1 \vee 2) = f(1) \vee f(2)$

lly for other Elements.

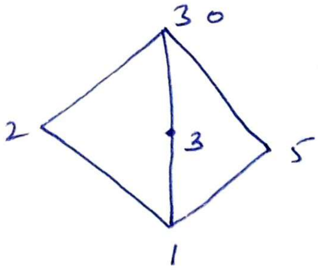
$\therefore$ f is isomorphism and

Lattices $(L, /)$ & $(P(A), \subseteq)$ are isomorphic.

4. Distributive Lattice :- A Lattice P is called distributive if $\forall\, a, b, c \in P$

i) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$
ii) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$ holds.

example 1. Given Hasse Diagram



$P = \{1, 2, 3, 5, 30\}$

Let $2, 3, 5 \in P$

L.H.S $2 \wedge (3 \vee 5) = 2 \wedge 30 = 2$

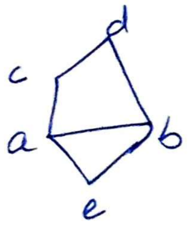
R.H.S $(2 \wedge 3) \vee (3 \wedge 5) = 1 \vee 1 = 1$

L.H.S $\neq$ R.H.S

It is not Distributive Lattice.

Note: · Every chain is a distributive lattice.
· A lattice is distributive iff it does not contain the 5- Element Pentagonal or the diamond lattice.

example 2.



Now $a \wedge (b \vee e) = a \wedge (b) = e$

$(a \wedge b) \vee (a \wedge e) = e \vee e = e$

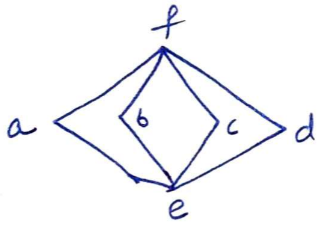
L.H.S = R.H.S.

$\parallel^{ly}$ $a \vee (b \wedge e) = a \vee e = a$

$(a \vee b) \wedge (a \vee e) = (d) \wedge a = a$

Hence this is distributive lattice.

example 3.



Now L.H.S $b \wedge (c \vee d) = b \wedge f = b$

R.H.S $(b \wedge c) \vee (b \wedge d) = e \vee e = e$

Hence LHS $\neq$ RHS

$\therefore$ It is not distributive lattice.

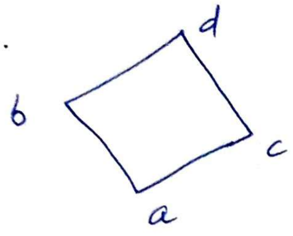
5. Complement Lattice :-

An Element 'x' in a bdd Lattice P is said to Complement of an Element $a \in P$ if

$$a \vee x = 1 \quad \& \quad a \wedge x = 0$$

1 = upper bound ; $1^c = 0$
0 = lower bound . $0^c = 1$

example 1.



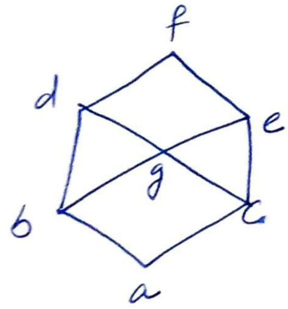
upper bound = d
Lower bound = a

$\therefore a^c = d$
$d^c = a$

Here $a \vee d = d = u.b$
$a \wedge d = a = l.b$

also $b \vee c = d = u.b$
$b \wedge c = a = l.b$ $\therefore b^c = c$
$c^c = b$

2.



upper bound = f
Lower bound = a

$\therefore a^c = f$
$f^c = a$

Here $b \wedge c = a = l.b$
but $b \vee c = e \neq u.b$ $\therefore b^c \neq c$

IIly $d \vee e = f = u.b$
but $d \wedge e = c \neq l.b$ $\therefore d^c \neq e$

$\therefore$ b, c, e, d has no Complement Element.